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Διερεύνηση της επιρροής της μικροδομής του υλικού στη δυναμική συμπεριφορά δικτυωμάτων

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ΠΕΡΙΛΗΨΗ

Στην παρούσα εργασία μελετάται η επίδραση της μικροδομής του υλικού στη δυναμική συμπεριφορά επίπεδων δικτυωμάτων. Δεδομένου ότι η κλασική ελαστικότητα δεν επαρκεί για την περιγραφή κατασκευών με διαστάσεις συγκρίσιμες με χαρακτηριστικά της μικροδομής τους, είναι απαραίτητη η εισαγωγή θεωριών ελαστικότητας ανωτέρας βαθμίδας. Στην παρούσα εργασία, χρησιμοποιείται η ελαστική θεωρία βαθμίδος της τροπής με την πλέον παραδοχή ότι η παράβλεψη των επιπλέον βαθμών ελευθερίας που αυτή εισάγει μας δίνει παραπλήσια αποτελέσματα με πολύ μικρή απόκλιση. Η ανάλυση πραγματοποιείται με τη μέθοδο των πεπερασμένων στοιχείων και χρήση γραμμικών συναρτήσεων σχήματος, αποφεύγοντας την υπολογιστική επιβάρυνση κατά τη χρήση των ακριβών συναρτήσεων σχήματος. Η επιρροή της θεωρίας βαθμίδος της τροπής εμφανίζεται με την εισαγωγή του χαρακτηριστικού μήκους στην διαφορική εξίσωση που διέπει το πρόβλημα. Σε όλες τις περιπτώσεις εξετάζεται η επιρροή της μικροδομής του υλικού και χρήσιμα συμπεράσματα προκύπτουν από τη μελέτη της δυναμικής συμπεριφοράς τους.

Λέξεις Κλειδιά: Ελαστική θεωρία βαθμίδος της τροπής, Μέθοδος των πεπερασμένων στοιχείων, Μικροδομή του υλικού, Επίπεδα δικτυώματα, Ιδιοσυχνότητες, Ιδιομορφές

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Investigation of the influence of material microstructure on the dynamic behavior of trusses

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ABSTRACT

In this study, the influence of the material's microstructure on the dynamic behavior of planar trusses is investigated. Since classical elasticity (CE) theory is insufficient to accurately describe the mechanical behavior of structures whose dimensions are comparable to those of their microstructure, higher-order elasticity theories are required. In this work, the strain gradient elasticity (SGE) theory is employed, under the assumption that neglecting the additional degrees of freedom it introduces yields similar results with minimal deviation. The analysis is conducted using the finite element method with linear shape functions, thus avoiding the computational burden associated with the exact shape functions. The impact of the gradient elasticity theory is reflected by introducing the characteristic length into the governing differential equation. In all cases, the influence of the material's microstructure is examined, and valuable conclusions are drawn regarding their dynamic behavior.

Keywords: Strain gradient elasticity, Finite Element Method, Material's Microstructure, Planar trusses, Eigenfrequencies, Mode shapes

1 INTRODUCTION

Strain gradient elasticity (SGE) theories enhance classical elasticity (CE) by incorporating higher-order strain terms, enabling the modeling of size-dependent mechanical behaviors. While CE assumes stress at a point is a function of local strain only, SGE theories account for strain gradients, making them particularly effective in capturing microstructural effects at nano- and micro-scales. Recently, there has been growing interest in applying these theories to meso- and macro-scale structures for refined analysis across broader engineering applications. In structural mechanics, especially for bars and beams, classical models like Euler–Bernoulli and Timoshenko often fail to capture size effects, leading to discrepancies in predicted behavior. SGE-based

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formulations address these shortcomings and have demonstrated improved accuracy in analyzing composite materials [1], pretwisted beams [2], cellular beams [3], and flexoelectric beams [4].

This study investigates the static and dynamic response of planar trusses using the finite element method within the simplified, engineering-oriented SGE framework developed by Sulem and Vardoulakis [5]. This SGE theory, as demonstrated by Giannakopoulos et al. [6], combines ease of implementation with improved validity, making it well-suited for practical applications. It has been successfully applied to bar tension [7], beam bending and stability [8], beam dynamics [9], cantilever bending and cracked bar problems [10], and Timoshenko beam analysis [11]. While most finite element (FE) formulations for strain gradient elasticity (SGE) have focused on bars and beams [12–17], planar trusses have received comparatively little attention. To date, only Akintayo [18] and Tsiatas et al. [19] addressed planar trusses, developing a two-node bar element. The latter formulation adopts the elastic node approach and introduces a gradient bar element with four degrees of freedom: two classical and two non-classical. The classical degrees of freedom represent axial displacements, whereas the non-classical ones correspond to axial strains.

In the present study, we extend the fundamental assumption of trusses—that members are pinned and therefore transmit only axial forces, not moments—by further postulating that axial strains at the member ends are also constrained. Specifically, the axial strains at the bar ends are prescribed a priori to be zero, thereby reducing the active degrees of freedom to the classical ones alone. Within this framework, the rigid node approach of SGE yields the same number of degrees of freedom as the CE theory, differing only in the modified stiffness to account for microstructural length effects. The method is straightforward to implement and produces static and dynamic results that closely match those from the elastic node formulation [19]. Additionally, comparisons with classical elasticity (CE) using consistent and lumped mass matrices provide valuable insights.

2 GRADIENT TRUSS ELEMENT: THE RIGID NODE APPROACH

Building on the work of Tsiatas et al. [19], the rigid node assumption is adopted by enforcing zero axial strains at the bar ends. Under this assumption, the stiffness matrix of a planar gradient truss element—characterized by length L_e, cross-sectional area A_e, modulus of elasticity E_e, and material density ρ_e—is expressed as:

$$\mathbf{k}_e^{PT} = \frac{E_e A_e}{L_e\left[1 - 2\kappa_e \tanh\left(\frac{1}{2\kappa_e}\right)\right]} \begin{bmatrix} 1 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 \\ -1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}, \tag{1}$$

where $\kappa_e = g/L_e$ being the ratio of the microstructural length, g, over the element's length, L_e.

To investigate the influence of the material microstructural length parameter g on the dynamic behavior of planar trusses, both the consistent and lumped mass matrices of the CE theory are employed. The consistent mass matrix in CE theory is expressed as:

$$\mathbf{m}_e^{CE,con} = \frac{\rho_e A_e L_e}{6}\begin{bmatrix} 2 & 0 & 1 & 0 \\ 0 & 2 & 0 & 1 \\ 1 & 0 & 2 & 0 \\ 0 & 1 & 0 & 2 \end{bmatrix}, \tag{2}$$

while the corresponding lumped mass matrix simplifies to:

$$\mathbf{m}_e^{CE,lum} = \frac{\rho_e A_e L_e}{2}\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}. \tag{3}$$

Finally, to incorporate the orientation of each element, a transformation matrix is employed. The direction cosines, $\cos\varphi^e$ and $\sin\varphi^e$, facilitate the conversion of local generalized nodal forces, $\mathbf{f}_S^e$, into the global coordinate system, $\bar{\mathbf{f}}_S^e$, as:

$$\mathbf{f}_S^e = \mathbf{R}_e^{PT}\bar{\mathbf{f}}_S^e, \tag{4}$$

where

$$\mathbf{R}_e^{PT} = \begin{bmatrix} \cos\varphi^e & \sin\varphi^e & 0 & 0 \\ -\sin\varphi^e & \cos\varphi^e & 0 & 0 \\ 0 & 0 & \cos\varphi^e & \sin\varphi^e \\ 0 & 0 & -\sin\varphi^e & \cos\varphi^e \end{bmatrix}, \tag{5}$$

is the transformation matrix of the truss element.

It is worth noting that Akintayo [18] also adopted the rigid node assumption, albeit by enforcing a different set of boundary conditions, which yields the following stiffness matrix:

$$\mathbf{k}_e^{PT,AK} = \frac{E_e A_e}{L_e\left[1 - \kappa_e \tanh\left(\frac{1}{\kappa_e}\right)\right]}\begin{bmatrix} 1 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 \\ -1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}. \tag{6}$$

3 STATICALLY DETERMINATE GRADIENT TRUSS

In this section, we analyze a statically determinate gradient truss (see Figure 1). Each truss member has a circular cross-section with a diameter of $D = 20$ mm and material properties of $E = 210$ GPa, $\rho = 7850$ kg/m. The truss is supported by pins at nodes 1 and 3, where all geometric boundary conditions are zero, while nodes 2 and 4 have all their degrees of freedom (DoF) unrestrained and free to move.

We begin with a static analysis under a load of $P = 100$ kN. The resulting displacements at various DoFs are summarized in Table 1 for different values of the microstructural length g. For comparison, displacements predicted by classical elasticity (CE) theory, obtained using a finite element (FE) formulation, show excellent agreement with strain gradient elasticity (SGE) theory results when $g = 0.001$ m. Overall, as g increases, displacements decrease, reflecting the stiffening effect of the truss.

To further highlight the deviations among the different theoretical approaches, Table 2 reports the displacements at the same DoF for $g = 0.5$ m. The results demonstrate that the present study provides values closer to those of the more accurate elastic node approach [19], compared to the formulation of Akintayo [18].

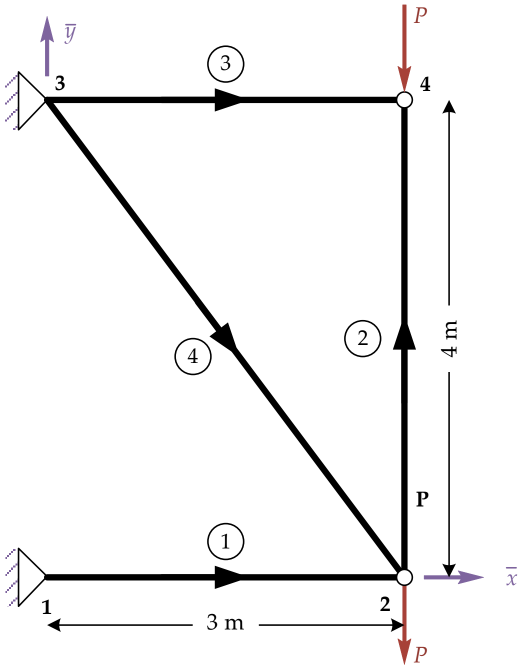

Figure 1: Geometry and loading of the gradient truss.

Table 1: Displacements (m) of the gradient truss at various DoF for various values of the microstructural length g (m).

DoF	FEM	$g = 0.001$	$g = 0.1$	$g = 0.2$	$g = 0.3$	$g = 0.4$	$g = 0.5$
u_3	-0.006821	-0.006816	-0.006366	-0.005911	-0.005457	-0.005004	-0.004559
u_4	-0.028799	-0.028787	-0.027511	-0.026223	-0.024934	-0.023647	-0.022366
u_8	-0.034863	-0.034847	-0.033271	-0.031679	-0.030088	-0.028498	-0.026915

Table 2: Displacements (m) at various DoF for $g = 0.5$ (m).

DOF	Rigid node Akintayo [18]	Rigid node Present study	Elastic node Tsiatas et al. [19]
u_3	-0.005684	-0.004559	-0.004735
u_4	-0.025578	-0.022366	-0.022880
u_8	-0.030884	-0.026915	-0.027671

The dynamic response of the gradient truss was also investigated. The first four natural frequencies, ω_n (rad/s), for various values of g are presented in Table 3, employing the consistent mass matrix of the CE theory. As g increases, ω_n generally rises, although the effect is less pronounced at lower frequencies. For further comparison, Table 4 presents the first four natural frequencies for $g = 0.5$ m. Again, the present formulation predicts the fundamental frequency more accurately than Akintayo's formulation [18].

Table 3: First four natural frequencies ω_n (rad/s) of the gradient truss for various values of the microstructural length g (m).

ω_n	FEM	$g = 0.001$	$g = 0.1$	$g = 0.2$	$g = 0.3$	$g = 0.4$	$g = 0.5$
1	577.875	578.009	591.759	606.682	622.778	640.186	659.012
2	1582.451	1582.942	1634.112	1691.407	1755.453	1827.476	1908.347
3	2084.362	2085.047	2156.380	2236.481	2326.274	2427.501	2541.321
4	2518.654	2519.274	2582.970	2652.549	2728.168	2810.750	2901.249

Table 4: First four natural frequencies ω_n (rad/s) of the gradient truss for $g = 0.5$ (m).

ω_n	Consistent mass matrix Akintayo [18]	Consistent mass matrix Present study	Consistent mass matrix Tsiatas et al. [19]
1	614.58	659.01	650.292
2	1722.51	1908.35	1718.49
3	2280.05	2541.32	2273.35
4	2689.55	2901.25	2564.37

4 CONCLUSIONS

This paper investigates the dynamic response of planar trusses using a simplified, engineering-oriented SGE theory. To this end, a two-node bar element with two degrees of freedom per node was developed. The proposed rigid-node formulation of the SGE theory maintains the same number of degrees of freedom as CE theory, differing only in the modified stiffness, which accounts for microstructural length-scale effects. The method is straightforward to implement and produces static and dynamic results that closely match those obtained using the elastic node formulation.

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